

CHANNEL ESTIMATION AND SYNCHRONIZATION FOR OFDM SYSTEM

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ABSTRACT

The OFDM system carries the message data on orthogonal subcarriers for parallel transmission, combating the distortion caused by the frequency selective channel or equivalently, the inter-symbol-interference in the multi-path fading channel. However, the advantage of the OFDM can be useful only when the orthogonality is maintained, thus In an OFDM system, the transmitter modulates the message bit sequence into PSK/QAM symbols, performs IFFT on the symbols to convert them into time domain signals, and sends them out through a (wireless) channel. The received signal is usually distorted by the channel characteristics. In order to recover the transmitted bits, the channel effect must be estimated and compensated in the receiver. First we will discuss synchronization techniques to manage the STO and CFO potential problems in OFDM systems, and the use of DFT technique for estimating the mobile radio channel response. Finally, we simulate the synchronization technique STO, and LS-linear, LS-spline and MMSE with DFT method for estimating the mobile radio channel parameters Bran A, thus a comparison of performance of LS-linear, LS-spline and MMSE with DFT method.

Keywords: OFDM, STO, CFO, LS-linear, LS-spline, MMSE, DFT, BRAN A.

INTRODUCTION

The advantage of the OFDM can be useful only when the orthogonality is maintained. In case the orthogonality is not sufficiently

warranted by any means, its performance may be degraded due to inter-symbol interference (ISI) and inter-channel

interference (ICI) [1]. Thus, the transmitted signal can be recovered by estimating the channel response just at each subcarrier. In general, the channel can be estimated by using a preamble or pilot symbols known to both transmitter and receiver, which employ various interpolation techniques to estimate the channel response of the subcarriers between pilot tones. In general, data signal as well as training signal, or both, can be used for channel estimation. In order to choose the channel estimation technique for the OFDM system under consideration, many different aspects of implementations, including the required performance, computational complexity and time-variation of the channel must be taken into account. In this article we discuss and simulate the synchronization OFDM system by STO technique, and the estimation mobile radio channel response BRAN A [2] by LS-linear, LS-spline and MMSE with DFT method.

I. SYNCHRONISATION FOR OFDM

In this part, we will analyze the effects of symbol time offset (STO) and carrier frequencies offset (CFO), and then discuss the synchronization techniques to handle the potential STO and CFO problems in OFDM systems. The received baseband signal under the presence of CFO and STO can be expressed as:

$$\begin{aligned} y[n] &= IDFT\{Y[n]\} \\ &= IDFT\{H[k]X[k] + Z[k]\} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{N} \sum_{k=0}^{N-1} H[k]X[k]e^{j2\pi(k+\varepsilon)(n+\delta)_N} \\ &\quad + z[n] \quad (1) \end{aligned}$$

Where $z[n] = IDFT\{Z[k]\}$ and ε, δ the normalized CFO and STO, respectively.

1. Estimation Techniques for STO

STO may cause not only phase distortion (that can be compensated by using an equalizer) but also ISI (that cannot be corrected once occurred) in OFDM systems. In order to warrant its performance, therefore, the starting point of OFDM symbols must be accurately determined by estimating the STO with a synchronization technique at the receiver. We discuss how to estimate the STO. In general, STO estimation can be implemented either in the time or frequency domain.

1.1 Time-Domain Estimation Techniques for STO

Consider an OFDM symbol with a cyclic prefix (CP) of N_G samples over T_G seconds and effective data of N_{sub} samples over T_{sub} seconds. In the time domain, STO can be estimated by using CP or training symbols. In the sequel, we discuss the STO estimation techniques with CP or training symbols.

a) STO Estimation Techniques Using Cyclic Prefix (CP)

Recall that CP is a replica of the data part in the OFDM symbol. It implies that CP and

the corresponding data part will share their similarities that can be used for STO estimation. The STO can be found by searching the point where the difference between two blocks of N_G samples within these two sliding windows is minimized [3], that is,

$$\hat{\delta} = \arg \min \left\{ \sum_{i=\delta}^{N_G-1+\delta} |y[n+i] - y[n+N+i]| \right\} \quad (2)$$

In spite of the simplicity of this technique, its performance can be degraded when CFO exists in the received signal. Another STO estimation technique, which can also deal with CFO, is to minimize the squared difference between a N_G -sample block (seized in window W_1) and the conjugate of another N_G -sample block (seized in window W_2) [4], shown as

$$\hat{\delta} = \arg \min \left\{ \sum_{i=\delta}^{N_G-1+\delta} (|y[n+i]| - |y^*[n+N+i]|)^2 \right\} \quad (3)$$

Another approach is to consider the correlation between those two blocks in W_1 and W_2 . Toward this end, a maximum-

likelihood estimation scheme can be applied to yield

$$\hat{\delta} = \operatorname{argmax} \left\{ \sum_{i=\delta}^{N_G-1+\delta} |y[n+i]y^*[n+N+i]| \right\} \quad (4)$$

Which corresponds to maximizing the correlation between a block of N_G samples (seized in window W_1) and another block of N_G samples (seized in window W_2). However, the performance of Equation (4) is degraded when CFO exists in the received signal. To deal with the CFO in the received signal, we utilize another ML technique that maximizes the log-likelihood function, given as:

$$\hat{\delta}_{ML} = \operatorname{argmax} \left\{ \sum_{i=\delta}^{N_G-1+\delta} \left[2(1 - \rho) \operatorname{Re}\{y[n+i]y^*[n+N+i]\} - \rho \sum_{i=\delta}^{N_G-1+\delta} |y[n+i] - y^*[n+N+i]| \right] \right\} \quad (5)$$

Where $\rho = SNR/(SNR + 1)$ [5]. We can also think of another ML technique that estimates both STO and CFO at the same

time as derived in [6]. In this technique, the STO is estimated as:

$$\hat{\delta}_{ML} = \operatorname{argmax}\{|\gamma[\delta]| - \rho\phi[\delta]\} \quad (6)$$

$$\gamma[m] = \sum_{n=m}^{m+L-1} y[n]y^*[n+N],$$

And

$$\phi[m] = \frac{1}{2} \sum_{n=m}^{m+L-1} \{|y[n]|^2 + |y[n+N]|^2\} \quad (7)$$

Using L to denote the actual number of samples used for averaging in windows. Taking the absolute value of the correlation [m], STO estimation in Equation (6) can be robust even under the presence of CFO.

b) STO Estimation Techniques Using Training Symbol

Training symbols can be transmitted to be used for symbol synchronization in the receiver. In contrast with CP, it involves overhead for transmitting training symbols, but it does not suffer from the effect of the multi-path channel. Once the transmitter sends the repeated training signals over two blocks within the OFDM symbol, the receiver attempts to find the CFO by maximizing the similarity between these two blocks of samples received within two sliding windows. The similarity between two sample blocks can be computed by an autocorrelation property of the repeated

training signal. As in the STO estimation technique using CP, for example, STO can be estimated by minimizing the squared difference between two blocks of samples received in W_1 and W_2 [7,8], such that

$$\hat{\delta} = \operatorname{argmin} \left\{ \sum_{i=\delta}^{\frac{N}{2}-1+\delta} \left| y[n+i] - y^* \left[n + \frac{N}{2} + i \right] \right|^2 \right\} \quad (8)$$

Or by maximizing the likelihood function [109], that is,

$$= \operatorname{argmax} \left\{ \frac{\left| \sum_{i=\delta}^{\frac{N}{2}-1+\delta} y[n+i]y^* \left[n + \frac{N}{2} + i \right] \right|^2}{\left| \sum_{i=\delta}^{\frac{N}{2}-1+\delta} y \left[n + \frac{N}{2} + i \right] \right|^2} \right\} \quad (9)$$

The difficulty in locating STO due to the flat interval can be handled by taking the average of correlation values over the length of CP [103], shown as

$$\hat{\delta} = \operatorname{argmax} \left(\frac{1}{N_G + 1} \sum_{m=-N_G}^i s[n+m] \right) \quad (10)$$

Where

$$s[n] = \frac{\left| \sum_{i=\delta}^{\frac{N}{2}-1+\delta} y[n+i]y^* \left[n + \frac{N}{2} + i \right] \right|^2}{\left(\frac{1}{2} \left| \sum_{i=\delta}^{\frac{N}{2}-1+\delta} y[n+i] \right|^2 \right)^2} \quad (11)$$

Now, the accuracy of STO estimation in Equation (9) can be further improved with

$\hat{\delta}$

$$= \operatorname{argmax}_{\hat{\delta}} \frac{\left| \sum_{i=\hat{\delta}}^{\frac{N}{4}-1+\delta} y \left[n + i + \frac{N}{4} + \frac{N}{2} m \right] \right|^2}{\left(\sum_{m=0}^1 \sum_{i=\hat{\delta}}^{\frac{N}{4}-1+\delta} \left| y \left[n + i + \frac{N}{4} + \frac{N}{2} m \right] \right|^2 \right)} \quad (12)$$

Another type of STO estimation technique is to use the cross-correlation between the training symbol and received signal, since the training symbol is known to the receiver. In this case, we do not need to use two sliding windows, W_1 and W_2 . In fact, only one sliding window which corresponds to the locally generated training symbol with a period of $T_{sub}/2$ is enough. Its performance can be degraded when CFO exists. In general, however, it provides better accuracy than the one using the auto-correlation property when the effect of CFO is not significant.

2. Estimation Techniques for CFO

Like STO estimation, CFO estimation can also be performed either in the time or the frequency domain.

2.1 Time-domain Estimation Techniques for CFO

or CFO estimation in the time domain, cyclic prefix (CP) or training symbol is used. Each of these techniques is described as below.

a) CFO Estimation Techniques Using Cyclic Prefix (CP)

With perfect symbol synchronization, a CFO

of ϵ results in a phase rotation of $2\pi\epsilon/N$ in the received signal. Under the assumption of negligible channel effect, the phase difference between CP and the corresponding rear part of an OFDM symbol (spaced N samples apart) caused by CFO ϵ is $2\pi N\epsilon/N = 2\pi\epsilon$. Then, the CFO can be found from the phase angle of the product of CP and the corresponding rear part of an OFDM symbol, for example, $\hat{\epsilon} = (1/2\pi)\operatorname{arg}\{y^*[n]y[n+N]\} = -1, -2, \dots, -N_g$. In order to reduce the noise effect, its average can be taken over the samples in a CP interval as

$$\hat{\epsilon} = \frac{1}{2\pi} \operatorname{arg} \left\{ \sum_{n=-N_G}^{-1} y^*[n]y[n+N] \right\} \quad (13)$$

Since the argument operation $\arg()$ is performed by using $\tan^{-1}()$, the range of CFO estimation in Equation (13) is $[-0.5, 0.5]$ so that $|\varepsilon| < 0.5$ and consequently, integral CFO cannot be estimated by this technique.

Note that $y^*[n]y[n+N]$ becomes real only when there is no frequency offset. This implies that it becomes imaginary as long as the CFO exists. In fact, the imaginary part of $y^*[n]y[n+N]$ can be used for CFO estimation [9]. In this case, the estimation error is defined as:

$$e_\varepsilon = \frac{1}{L} \sum_{n=1}^L \text{Im}\{y^*[n]y[n+N]\} \quad (14)$$

Where L denotes the number of samples used for averaging. Note that the expectation of the error function in Equation (14) can be approximated as

$$E\{e_\varepsilon\} = \frac{\sigma_d^2}{N} \sin\left(\frac{2\pi\varepsilon}{N}\right) \sum_{k \text{ corresponding to useful carriers}}^L |H_k|^2 \approx K\varepsilon \quad (15)$$

Where σ_d^2 is the transmitted signal power, H_k is the channel frequency response of the k th subcarrier, and K is a term that comprises transmit and channel power.

b) CFO Estimation Techniques Using Training Symbol

We have seen that the CFO estimation technique using CP can estimate the CFO only within the range $\{|\varepsilon| \leq 0.5\}$. Since CFO can be large at the initial synchronization stage, we may need estimation techniques that can cover a wider CFO range. The range of CFO estimation can be increased by reducing the distance between two blocks of samples for correlation. This is made possible by using training symbols that are repetitive with some shorter period. Let D be an integer that represents the ratio of the OFDM symbol length to the length of a repetitive pattern. Let a transmitter send the training symbols with D repetitive patterns in the time domain, which can be generated by taking the IFFT of a comb-type signal in the frequency domain given as:

$$X[k] = \begin{cases} A_m, & \text{if } k = D \cdot i, \quad i = 0, 1, \dots, (N/D - 1) \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

Where A_m represents an M -ary symbol and N/D is an integer. As $x[n]$ and $x[n+N/D]$ are identical (i.e., $y^*[n]y[n+N/D] = |y[n]|^2 e^{j\pi\varepsilon}$), a receiver can make CFO estimation as follows [7,8]:

$$\hat{\varepsilon} = \frac{1}{2\pi} \arg \left\{ \sum_{n=-N_G}^{-1} y^*[n]y[n+N/D] \right\} \quad (17)$$

The CFO estimation range covered by this technique is $\{|\varepsilon| \leq D/2\}$, which becomes wider as D increases. Note that the number

of samples for the computation of correlation is reduced by $1/D$, which may degrade the MSE performance. In other words, the increase in estimation range is obtained at the sacrifice of MSE (mean square error) performance. As the estimation range of CFO increases, the MSE performance becomes worse. By taking the average of the estimates with the repetitive patterns of the shorter period as

$$\hat{\varepsilon} = \frac{1}{2\pi} \arg \left\{ \sum_{m=0}^{D-2} \sum_{n=0}^{N/D-1} y^*[n + mN/D]y[n + (m+1)N/D] \right\} \quad (18)$$

The MSE performance can be improved without reducing the estimation range of CFO.

2.2 Frequency-Domain Estimation Techniques for CFO

If two identical training symbols are transmitted consecutively, the corresponding signals with CFO of ε are related with each other as follows:

$$\begin{aligned} y_2[n] &= y_1[n]e^{j2\pi N\varepsilon/N} \leftrightarrow Y_2[k] \\ &= Y_1[k]e^{j2\pi\varepsilon} \end{aligned} \quad (19)$$

Using the relationship in Equation (19), the CFO can be estimated as

$$\hat{\varepsilon} = \frac{1}{2\pi} \tan^{-1} \left\{ \frac{\sum_{k=0}^{N-1} \text{Im}[Y_1^*[k]Y_2[k]] / \sum_{k=0}^{N-1} \text{Re}[Y_1^*[k]Y_2[k]]]}{\sum_{k=0}^{N-1} \text{Re}[Y_1^*[k]Y_2[k]]} \right\} \quad (20)$$

Which is a well-known approach by Moose [10]. Although the range of CFO estimated by Equation (20) is $|\varepsilon| \leq 1/2$. In this case, Equation (5.30) is applied to the subcarriers with non-zero value and then, averaged over the subcarrier.

II. CHANNEL ESTIMATION

The transmitted signal can be recovered by estimating the channel response just at each subcarrier. In general, the channel can be estimated by using a preamble or pilot symbols known to both transmitter and receiver, which employ various interpolation techniques to estimate the channel response of the subcarriers between pilot tones. In general, data signal as well as training signal, or both, can be used for channel estimation. In order to choose the channel estimation technique for the OFDM system under consideration, many different aspects of implementations, including the required performance, computational complexity and time-variation of the channel must be taken into account.

1. Training Symbol-Based Channel Estimation

Training symbols can be used for channel estimation, usually providing a good

performance. However, their transmission efficiencies are reduced due to the required overhead of training symbols such as preamble or pilot tones that are transmitted in addition to data symbols. The least-square (LS) and minimum-mean-square-error (MMSE) techniques are widely used for channel estimation when training symbols are available [11–14,15,16]. We assume that all subcarriers are orthogonal. Then, the training symbols for N subcarriers can be represented by the following diagonal matrix:

$$\mathbf{X} = \begin{bmatrix} X[0] & 0 & \dots & 0 \\ 0 & X[1] & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & X[N-1] \end{bmatrix}$$

Where $X[k]$ denotes a pilot tone at the k^{th} subcarrier, with $E\{X[k]\} = 0$ and $Var\{X[k]\} = \sigma_x^2$, $k = 0, 1, 2, \dots, N - 1$. Note that \mathbf{X} is given by a diagonal matrix, since we assume that all subcarriers are orthogonal. Given that the channel gain is $H[k]$ for each subcarrier k , the received training signal $Y[k]$ can be represented as:

$$\mathbf{Y} \triangleq \begin{bmatrix} Y[0] \\ Y[1] \\ \vdots \\ Y[N-1] \end{bmatrix}$$

$$\begin{aligned} &= \\ &\begin{bmatrix} X[0] & 0 & \dots & 0 \\ 0 & X[1] & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & X[N-1] \end{bmatrix} \begin{bmatrix} H[0] \\ H[1] \\ \vdots \\ H[N-1] \end{bmatrix} + \\ &\begin{bmatrix} Z[0] \\ Z[1] \\ \vdots \\ Z[N-1] \end{bmatrix} = \mathbf{XH} + \mathbf{Z} \end{aligned} \quad (21)$$

Where \mathbf{H} is a channel vector given as $\mathbf{H} = [H[0], H[1], \dots, H[N-1]]^T$ and \mathbf{Z} is a noise vector given as $\mathbf{Z} = [Z[0], Z[1], \dots, Z[N-1]]$ with $E\{Z[k]\} = 0$ and $Var\{Z[k]\} = \sigma_z^2$, $k = 0, 1, 2, \dots, N - 1$. In the following discussion, let $\hat{\mathbf{H}}$ denote the estimate of channel \mathbf{H} .

1.1 LS Channel Estimation

The least-square (LS) channel estimation method finds the channel estimate $\hat{\mathbf{H}}$ in such a way that the following cost function is minimized:

$$\begin{aligned} J(\hat{\mathbf{H}}) &= \|\mathbf{Y} - \mathbf{X}\hat{\mathbf{H}}\|^2 \\ &= (\mathbf{Y} - \mathbf{X}\hat{\mathbf{H}})^H(\mathbf{Y} - \mathbf{X}\hat{\mathbf{H}}) \\ &= \mathbf{Y}^H\mathbf{Y} - \mathbf{Y}^H\mathbf{X}\hat{\mathbf{H}} - \hat{\mathbf{H}}^H\mathbf{X}^H\mathbf{Y} + \hat{\mathbf{H}}^H\mathbf{X}^H\mathbf{X}\hat{\mathbf{H}} \end{aligned} \quad (22)$$

By setting the derivative of the function with respect to $\hat{\mathbf{H}}$ to zero,

$$\text{We have } \mathbf{X}^H\mathbf{X}\hat{\mathbf{H}} = \mathbf{X}^H\mathbf{Y}, \quad (23)$$

which gives the solution to the LS channel estimation as:

$$\hat{\mathbf{H}}_{LS} = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbf{Y} = \mathbf{X}^{-1} \mathbf{Y}$$
(24)

Let us denote each component of the LS channel estimate $\hat{\mathbf{H}}_{LS}$ by $\hat{H}_{LS}[k]$, $k=0, 1, 2, \dots, N-1$.

Since \mathbf{X} is assumed to be diagonal due to the ICI-free condition, the LS channel estimate $\hat{\mathbf{H}}_{LS}$ can be written for each subcarrier as:

$$\hat{H}_{LS}[k] = \frac{Y[k]}{X[k]}, \quad k = 0, 1, 2, \dots, N - 1$$
(25)

The mean-square error (MSE) of this LS channel estimate is given as

$$\begin{aligned} MSE_{LS} &= E\{(H - \hat{H}_{LS})^H (H - \hat{H}_{LS})\} \\ &= E\{(H - X^{-1}Y)^H (H - X^{-1}Y)\} \\ &= E\{(X^{-1}Z)^H (X^{-1}Z)\} \\ &= E\{Z^H (XX^H)^{-1} Z\} \\ &= \frac{\sigma_z^2}{\sigma_x^2} \end{aligned}$$
(26)

Note that the MSE in Equation (26) is inversely proportional to the SNR $\frac{\sigma_x^2}{\sigma_z^2}$, which implies that it may be subject to noise enhancement, especially when the channel is in a deep null. Due to its simplicity,

however, the LS method has been widely used for channel estimation.

1.2 MMSE Channel Estimation

Consider the LS solution in Equation (24), $\hat{\mathbf{H}}_{LS} = \mathbf{X}^{-1} \mathbf{Y} \triangleq \tilde{\mathbf{H}}$. Using the weight matrix \mathbf{W} , define

$\hat{\mathbf{H}} \triangleq \mathbf{W} \tilde{\mathbf{H}}$, which corresponds to the MMSE estimate. MSE of the channel estimate $\hat{\mathbf{H}}$ is given as

$$J(\hat{H}) = E\{\|e\|^2\} = E\{\|H - \hat{H}\|^2\}$$
(27)

Then, the MMSE channel estimation method finds a better (linear) estimate in terms of \mathbf{W} in such a way that the MSE in Equation (27) is minimized. The orthogonality principle states that the estimation error vector $\mathbf{e} = \mathbf{H} - \hat{\mathbf{H}}$ is orthogonal to $\tilde{\mathbf{H}}$, such that

$$\begin{aligned} E\{\mathbf{e} \tilde{\mathbf{H}}^H\} &= E\{(\mathbf{H} - \hat{\mathbf{H}}) \tilde{\mathbf{H}}^H\} \\ &= E\{(\mathbf{H} - \mathbf{W} \tilde{\mathbf{H}}) \tilde{\mathbf{H}}^H\} \\ &= E\{\mathbf{H} \tilde{\mathbf{H}}^H\} - \mathbf{W} E\{\tilde{\mathbf{H}} \tilde{\mathbf{H}}^H\} \\ &= \mathbf{R}_{H \tilde{H}} - \mathbf{W} \mathbf{R}_{\tilde{H} \tilde{H}} = 0 \end{aligned}$$
(28)

Where \mathbf{R}_{AB} is the cross-correlation matrix of $N \times N$ matrices \mathbf{A} and \mathbf{B} (i.e., $\mathbf{R}_{AB} = E[\mathbf{AB}^H]$), and $\tilde{\mathbf{H}}$ is the LS channel estimate given as

$$\tilde{\mathbf{H}} = \mathbf{X}^{-1} \mathbf{Y} = \mathbf{H} + \mathbf{X}^{-1} \mathbf{Z}$$
(29)

Solving Equation (28) for \mathbf{W} yields

$$\mathbf{W} = \mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}} \mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}}^{-1} \quad (30)$$

Where $\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}}$ is the autocorrelation matrix of $\tilde{\mathbf{H}}$ given as

$$\begin{aligned} \mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}} &= \mathbb{E}\{\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\} \\ &= \mathbb{E}\{\mathbf{I} - \mathbf{C}\mathbf{C}^H(\mathbf{I} - \mathbf{B}\mathbf{B}^H)\} \\ &= \mathbb{E}\{(\mathbf{I} + \mathbf{C}\mathbf{C}^H)(\mathbf{I} + \mathbf{B}\mathbf{B}^H)\} \\ &= \mathbb{E}\{\mathbf{I}\mathbf{I}^H + \mathbf{C}\mathbf{C}^H\mathbf{I}^H \\ &\quad + \mathbf{C}\mathbf{C}^H(\mathbf{I} - \mathbf{B}\mathbf{B}^H) \\ &\quad + \mathbf{B}\mathbf{B}^H\mathbf{C}\mathbf{C}^H(\mathbf{I} - \mathbf{B}\mathbf{B}^H)\} \\ &= \mathbb{E}\{\mathbf{I}\mathbf{I}^H\} + \mathbb{E}\{\mathbf{C}\mathbf{C}^H\mathbf{I}^H(\mathbf{I} - \mathbf{B}\mathbf{B}^H)\} \\ &= \mathbb{E}\{\mathbf{I}\mathbf{I}^H\} + \mathbb{E}\left\{\mathbf{C}\mathbf{C}^H\left(\mathbf{I} - \frac{\mathbf{B}^H\mathbf{B}}{\mathbf{B}^H\mathbf{B}}\mathbf{B}\mathbf{B}^H\right)\right\} \end{aligned} \quad (31)$$

And $\mathbf{R}_{\mathbf{C}\tilde{\mathbf{H}}}$ is the cross-correlation matrix between the true channel vector and temporary channel estimate vector in the frequency domain. Using Equation (31), the MMSE channel estimate follows as

$$\begin{aligned} \hat{\mathbf{H}} &= \mathbf{C}\tilde{\mathbf{H}} = \mathbf{R}_{\mathbf{C}\tilde{\mathbf{H}}}\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}}^{-1}\tilde{\mathbf{H}} \\ &= \mathbf{R}_{\mathbf{C}\tilde{\mathbf{H}}}\left(\mathbf{I} + \frac{\mathbf{B}^H\mathbf{B}}{\mathbf{B}^H\mathbf{B}}\mathbf{B}\mathbf{B}^H\right)^{-1}\tilde{\mathbf{H}} \end{aligned} \quad (32)$$

The elements of $\mathbf{R}_{\mathbf{C}\tilde{\mathbf{H}}}$ and $\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}}$ in Equation (6.14) are

$$\begin{aligned} \mathbb{E}\{\mathbf{C}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\} &= \mathbb{E}\{\mathbf{C}\mathbf{C}^H\mathbf{I}^H\} \\ &= \mathbb{E}_k[\mathbf{C}] \mathbb{E}_l[\mathbf{I}] \mathbb{E}_k[\mathbf{C}]^H \end{aligned} \quad (33)$$

Where k and l denote the subcarrier (frequency) index and OFDM symbol (time) index, respectively. In an exponentially-decreasing multipath PDP (Power Delay Profile), the frequency-domain correlation $\mathbb{E}_k[\mathbf{C}]$ is given as

$$\mathbb{E}_k[\mathbf{C}] = \frac{1}{1 + \frac{1}{2\pi\Delta f\Delta t}\Delta f} \quad (34)$$

where $\Delta f = 1/\Delta t$ is the subcarrier spacing for the FFT interval length of N_{FFT} . Meanwhile, for a fading channel with the maximum Doppler frequency Δf and Jake's spectrum, the time-domain correlation $\mathbb{E}_t[\mathbf{I}]$ is given as

$$\mathbb{E}_t[\mathbf{I}] = J_0(2\pi\Delta f\Delta t) \quad (35)$$

Where $J_0(x) = J_0 + J_1$ for guard interval time of Δt and $J_0(x)$ is the first kind of 0th-order Bessel function. Note that $J_0[0] = J_0(0) = 1$.

To estimate the channel for data symbols, the pilot subcarriers must be interpolated. Popular interpolation methods include linear interpolation, second-order polynomial interpolation, and cubic spline interpolation [13,14,17,18].

2. DFT-Based Channel Estimation

The DFT-based channel estimation technique has been derived to improve the performance of LS or MMSE channel estimation by eliminating the effect of noise outside the maximum channel delay. Let $\hat{h}[k]$ denote the estimate of channel gain at the k th subcarrier, obtained by either LS or MMSE channel estimation method. Taking the IDFT of the channel estimate $\{\hat{h}[k]\}_{k=0}^{L-1}$,

$$\begin{aligned} \text{IDFT}\{\hat{h}[k]\} &= \sum_{k=0}^{L-1} \hat{h}[k] e^{-j\frac{2\pi}{L} k n} + z[n] \triangleq \hat{h}[n], \\ n &= 0, 1, 2, \dots, L-1 \end{aligned} \quad (36)$$

Where $z[n]$ denotes the noise component in the time domain. Ignoring the coefficients $\hat{h}[k]$ that contain the noise only, define the coefficients for the maximum channel delay L as

$$\begin{aligned} \hat{h}_{\text{max}}[n] &= \sum_{k=0}^{L-1} \hat{h}[k] e^{-j\frac{2\pi}{L} k n}, \\ n &= 0, 1, 2, \dots, L-1 \end{aligned} \quad (37)$$

And transform the remaining L elements back to the frequency domain as follows [19–22]:

$$\hat{h}_{\text{max}}[n] = \text{DFT}\{\hat{h}_{\text{max}}[k]\} \quad (38)$$

III. SIMULATION RESULTS

1. STO Estimation

In the simulation we test the performance of CP-based STO estimation with the Number of samples corresponding to STO is $n\text{STO}=[-3 -3 3 3]$ and CFO= [0.00 0.50].

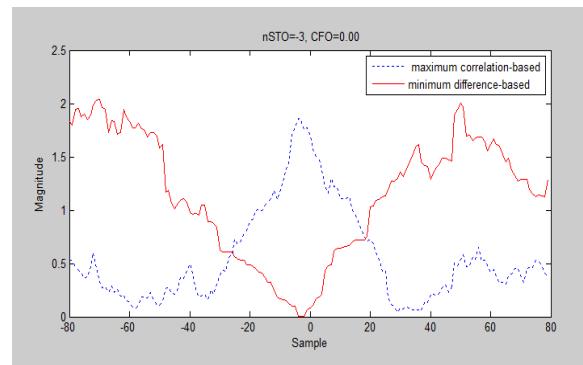


Fig.1: Performance of CP-based STO estimation for $n\text{STO}=-3$ and $\text{CFO}=0.00$: maximum correlation-based vs minimum difference-based estimation.

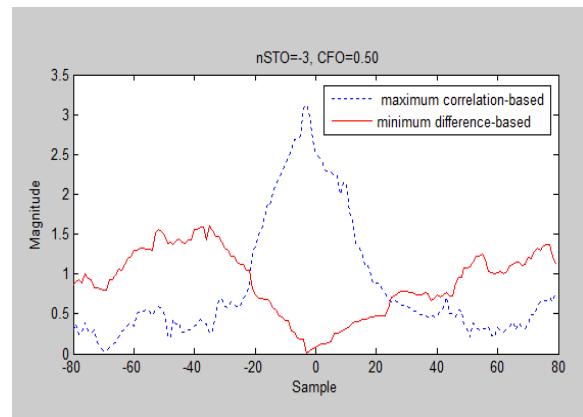


Fig.2: Performance of CP-based STO estimation for $n\text{STO}=-3$ and $\text{CFO}=0.50$: maximum correlation-based vs minimum difference-based estimation.

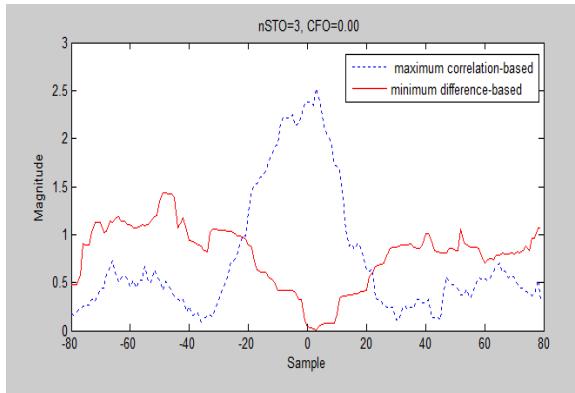


Fig.3: Performance of CP-based STO estimation for $nSTO=3$ and $CFO=0.00$: maximum correlation-based vs minimum difference-based estimation.

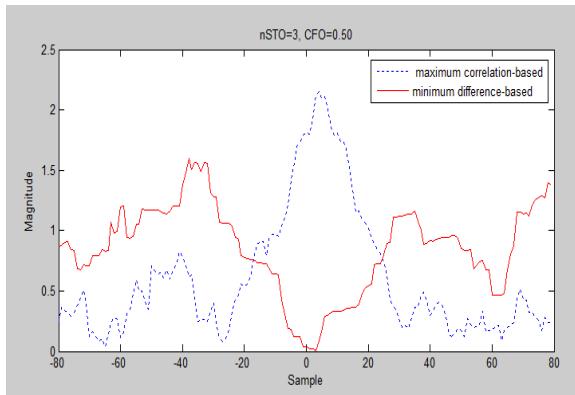


Fig.4: Performance of CP-based STO estimation for $nSTO=3$ and $CFO=0.50$: maximum correlation-based vs minimum difference-based estimation.

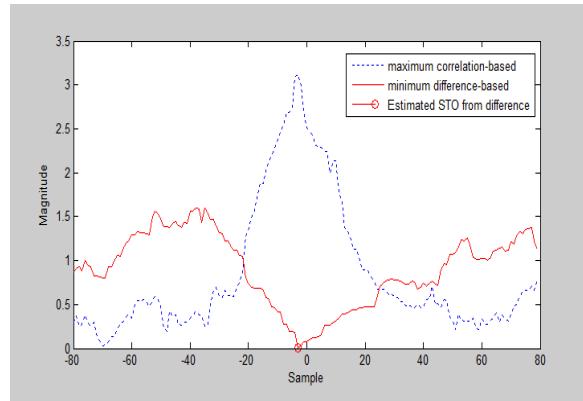


Fig.5: Performance of STO estimation from difference between CP (cyclic prefix) and rear part of OFDM symbol

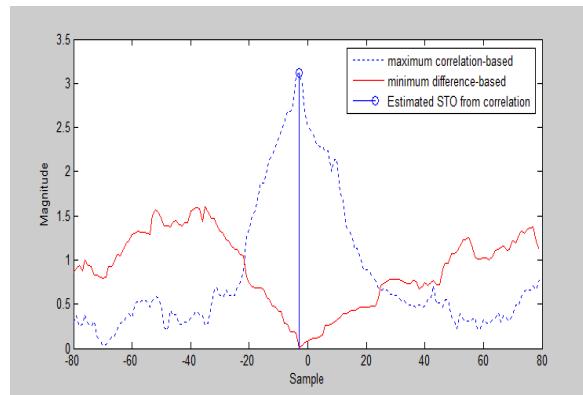


Fig.5: Performance of STO estimation from difference between CP (cyclic prefix) and rear part of OFDM symbol

the results obtained by STO estimation using CP, in which CFO is located at the point of minimizing the difference between the sample blocks of CP and that of data part or maximizing their correlation.

2. Channel Estimation

In this part, we will test the performance of LS-linear, LS-spline and MMSE with DFT

method for estimating the mobile radio channel parameters Bran A.

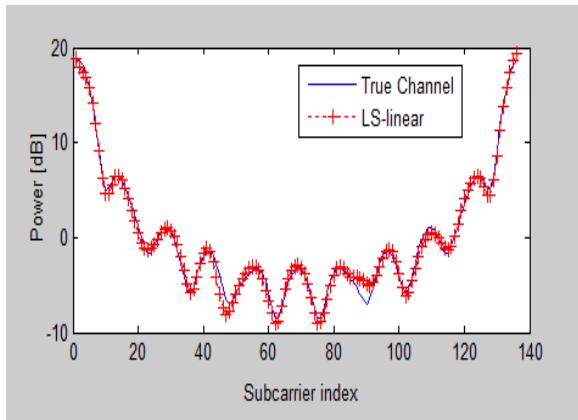


Fig.6: LS-Linear estimate without DFT technique.

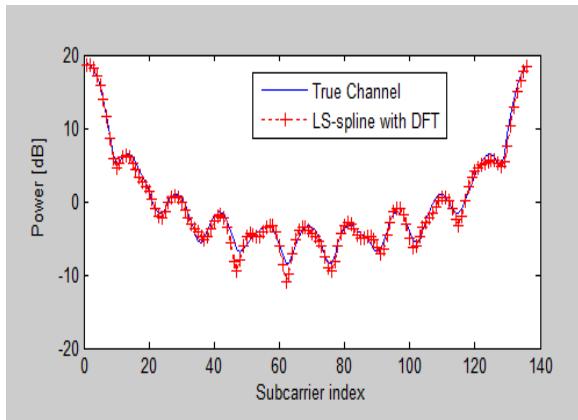


Fig.7: DFT-based LS-Linear estimate.

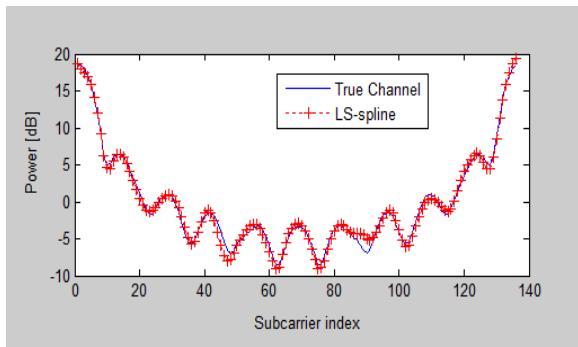


Fig.8: LS-Spline estimate without DFT technique.

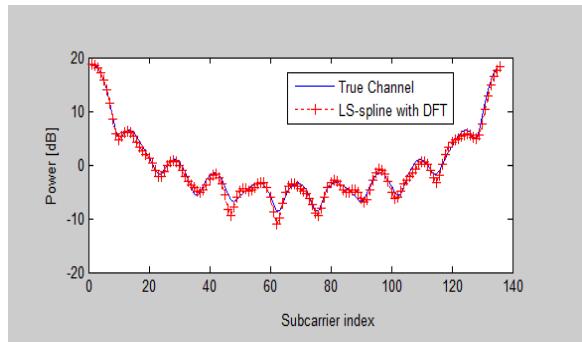


Fig.9: DFT-based LS-spline estimate.

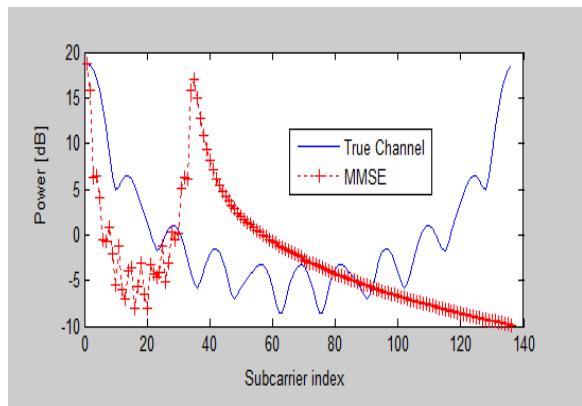


Fig.10: MMSE estimate without DFT technique.

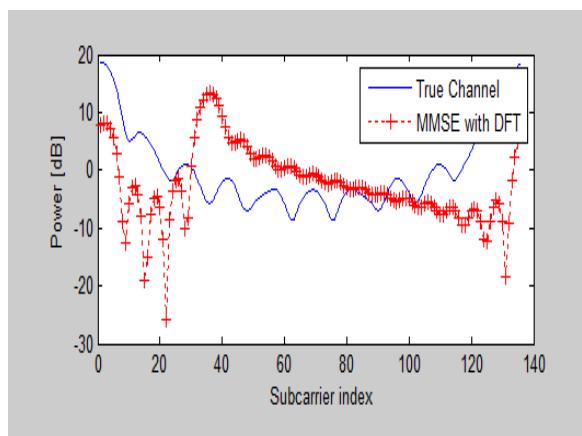


Fig.11: DFT-based MMSE estimate.

the results of experiments illustrates the mobile radio channel estimates A bran obtained using different types of channel estimation methods with or without technical DFT discussed in the foregoing. Comparing Figures 6, 8, and 10 with Figures 7, 9, and 11 reveals that the DFT-based channel estimation method improves the performance of channel estimation. Also, comparing Figures 6 and 8 with Figure 10, it is clear that the LS estimation shows better performance than the MMSE estimation.

IV. CONCLUSION

In this article, we presented synchronization for OFDM system by STO method and the estimation of mobile radio channel bran A by the LS and MMSE methods with DFT technology. Simulation results show that the STO estimation using CP is located at the point of minimizing the difference between the sample blocks of CP and that of data part or maximizing their correlation. Thus, the method of mobile radio channel estimation based on DFT improves the performance of the channel estimation, also the LS estimation shows better performance than the MMSE estimation.

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