

PRECISE LOCAL GEOID DEFINITION CASE STUDY: NISYROS ISLAND IN GREECE

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ABSTRACT

The global geoid model EGM08 consists nowadays one of the best tools for quick transformation of geometric heights which provided by GNSS measurements to orthometric ones. Nevertheless there are some areas worldwide, where the adaptation of EGM08 is not satisfactory due to large terrain fluctuations (high mountains, spread islands etc.) or due to strong gravity anomalies.

Moreover, the unification of vertical datums between the islands and mainland in Greece as well as between adjacent countries remains a main goal which is mainly supported by satellite gravity missions, e.g., the CHAMP, GRACE and GOCE satellites in order to produce accurate and reliable GGMs.

Thus, the determination of accurate (under cm) orthometric heights via the geometric ones is required by the demanding infrastructure and monitoring projects.

It is well known that geometric heights are easily obtained by GNSS measurements, while the orthometric ones are extremely demanding in time and staff in order to be determined by some mm accuracy.

This study applies a simultaneous measurement project of orthometric and geometric height differences in order to assess in terms of time and cost, the calculation of a precise (under cm) local geoid on Nisyros island- Greece.

Nisyros Island is located at the southeast edge of the Aegean Sea. A 13 benchmarks (BMs) network was properly spread on the island. GNSS measurements were carried out by the relative static positioning method. The orthometric height differences between BMs were measured by the method of Accurate Trigonometric Heighting. The orthometric and geometric height of each benchmark (BM) was determined with $\pm 6\text{mm}$ and $\pm 10\text{mm}$ respectively by applying least square adjustments. The best adaptation equation is determined for the local geoid undulation. Additionally the corresponding values of the geoid undulation N were calculated by the EGM08

as well as a reduction equation. Moreover the zero-level geopotential value (W_0) of the island, has been determined and be compared to the corresponding ones of the neighboring to Nisyros Hellenic islands and the Greek Vertical Datum (GVD).

Keywords: local geoid model, Accurate Trigonometric Heighting, reduction equation for EGM08, Local Vertical Datum, zero-level geopotential value, Nisyros Island

I. INTRODUCTION

The Global positioning system provides ellipsoidal (geometric) heights, which have no physical meaning. For this reason they aren't used in engineering and infrastructure applications. Thus the main request of the geodetic community is the accurate transformation of the acquired geometric heights to orthometric ones. The Global geoid model EGM08 contributes a lot to this effort as includes enormous quantity of several kinds of data (altimetry, gravimetric, leveling). It provides global geoid estimation of the order of ± 5 to ± 10 cm over areas covered with high quality of gravity data. The model includes gravity anomalies according to a global 5 arc-minute grid and reaches spherical harmonic degree of order 2159 which corresponds to a grid size of approximately 7km on the earth surface [1],[2],[3].

Also reliable geoid models may be derived nowadays from gravity data obtained from satellite gravity missions, e.g., the CHAMP, GRACE and GOCE satellites. These GGMs provide adequate accuracies for a large number of geodetic applications [4], [5]. Additionally, GOCE data support the determination of the zero-level geopotential value towards the unification of Local

Vertical Datums (LVD) to a global one. [6], [7]

However in some areas, where there is lack of data or large scale topographic discontinuities, large discrepancies of half a meter or more are noticed [8]. Also for small areas under the resolution of the EGM08 the provided data may be unreliable. In these cases the comparison of a well fitted local model with the global ones may give useful information for the examined area and the opportunity to use the EGM08 or GGMs by following the calculated reduction equation.

The successfulness of the scientific researches is to determine geoid heights at cm level as this is required by the modern infrastructure and constructions [9],[10],[11],[12],[13],[14],[15].

A well-known method for the calculation of the local geoids model, which is also used for the evaluation of the above mentioned global models, is the GPS/leveling data processing. This demands both geometric and orthometric heights at adequate number of control points at the interest area in order to calculate satisfactory local geoid heights by sub centimeter accuracy. This presupposes to obtain both the geometric

and orthometric heights of the control points with some mm accuracy. Thus the geoid heights as well as their uncertainties are calculated by using the well known equations 1.

$$N_i = H_i - h_i \text{ and } \sigma_{N_i} = \sqrt{\sigma_{H_i}^2 + \sigma_{h_i}^2} \quad (1)$$

In practice, this procedure is the approximation by using measured GNSS and leveling height differences. So the geoid heights N are used for a model creation in the interest area. The geodetic coordinates (ϕ , λ) or the local plan coordinates (x , y) are used for the adjustment[16].

Many researches have been carried out on the local geoid determination by using conventional, modern and artificial neural networks methods [17], [18], [19], [20], [21], [15]. Well known techniques as the bilinear interpolation, polynomial regression, triangulation, nearest-neighbor interpolation [22], [8] and Artificial Neural networks (ANNs) have recently be used for this purpose with success [23], [24], [25], [26]. The method to be used for the modeling depends on the number of control points, the degree of freedom and the size of the area as far as the statistical tests are satisfied [27], [28], [29].

In the aforementioned works usually the used geometric height differences come out by direct GNSS measurements between the BMs as the orthometric ones, due to the difficulty in their acquisition, are provided

by different time period campaigns. Usually they are of less accuracy and reliability.

The aim of this work is to investigate the possibility, the time and the cost needed in order to perform a simultaneous orthometric and geometric height differences measurement project for the determination of sub centimeter local geoid.

II. THE STUDY AREA AND THE DATA ACQUISITION

Nisyros Island is located at the southeast edge of the Aegean Sea (figure 1a) covering an area of 41Km². It has almost round shape, with radius 3.6Km. Nisyros was created by the volcano eruptions, which is situated almost at its center.



Figure 1a: The Nisyros' Island

A network of 13 BMs was established (figure 1b), on the island. The benchmarks are distributed at a grid of 1km to 4Km in

order to cover the examined area. Special attention was paid for their accessibility. Twenty eight connections were formed between them.

Thus for the geometric heights acquisition 28 baselines of 1.5km to 6Km were measured by using the GNSS static positioning method. Two Trimble 5800 double frequency GNSS receivers are used [30].

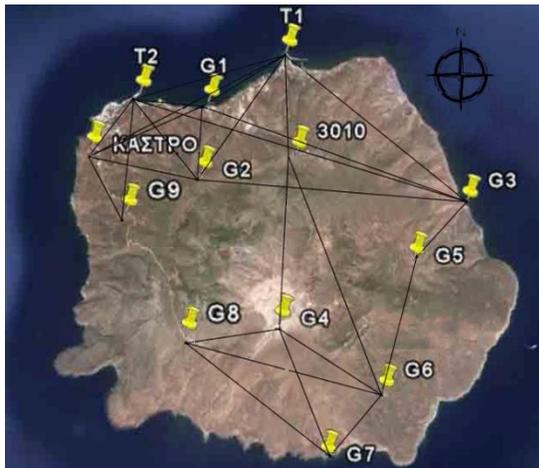


Figure 1b: The geodetic network

The occupation time for each baseline ranges between 1 to 1.5 hours. The net time for the GNSS measurements was about 40 hours as the total needed time including the transportations of the instrumentation is about 5 days, by a crew of at least two persons.

The 28 orthometric height differences (ΔH) between the same BMs were measured by using the accurate trigonometric heighting method [31], [32]. This method uses the trigonometric leveling method by

simultaneous and reciprocal observations of zenith angles and distances between the points. Thus the error of the refraction is eliminated and also the instrument and target heights aren't needed. The use of this method for height difference measurement ensures errors of some mm for distances up to 5 Km. The total stations Leica TCR1201⁺ and TM30 are used which provide $\pm 1''$ and $\pm 0.5''$ angle accuracy respectively and ± 1 mm distance measurement accuracy [33], [34]. The time needed for each height difference measurement ranges between 1 to 3 hours. This depends on the distance between the points, the total number of the intermediate instrument stations and the time for the instrumentation transportation.

The total time for the network measurement reaches 10 days, by a crew of at least three persons.

III. DATA PROCESSING

The BM T1 which is a cement pillar of the Hellenic Military Geographic Service (HMGS), was decided to be the fix point of the network. So the orthometric height of T1 is provided by the national network adjustment[35]. The geometric height of T1 is provided, relative to the GRS80 ellipsoid, by the Hellenic positionig system HEPOS, when a six month campain was carried out on 2007 in order to determine reliable GNSS measurements and coordinates for 2470 stations in Greece for the system completion[36].

The appropriate reductions are implemented to both T1 heights for their transformation to the zero-tide system in order to be compatible with the Local Vertical Datum (LVD) calculation which will follow. For the geometric height the reduction [37], [38] is given by the equation 2

$$h^{ZT} = h^{FT} + \ell \cdot (0.099 - 0.296 \cdot \sin^2\varphi) \quad (2)$$

where $\ell = 0.62$ and h^{FT} and h^{ZT} are the geometric height of the tide free and zero-tide reference ellipsoid. This negative correction equals to -4 mm .

Moreover the orthometric height is referred to measurements implemented by the HMGS in Greece where no tidal corrections were performed [39] thus the equation 3 is used

$$H^{ZT} = H^{MT} + (0.099 - 0.296 \cdot \sin^2\varphi) \quad (3)$$

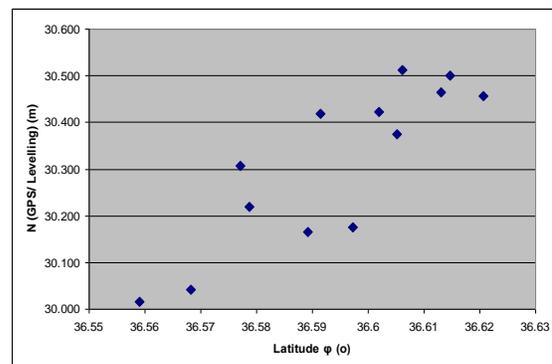
where H^{ZT} and H^{MT} are the orthometric heights of the zero-tide geoid and the mean tide geoid respectively. This negative correction equals to -6 mm.

Separate least square adjustment was applied for each network (orthometric and geometric) with 12 unknown heights and 16 degrees of freedom. Thus the orthometric and geometric heights of each benchmark are determined with uncertainties, which range between ± 3 mm to ± 6 mm and ± 6 mm to ± 10 mm respectively.

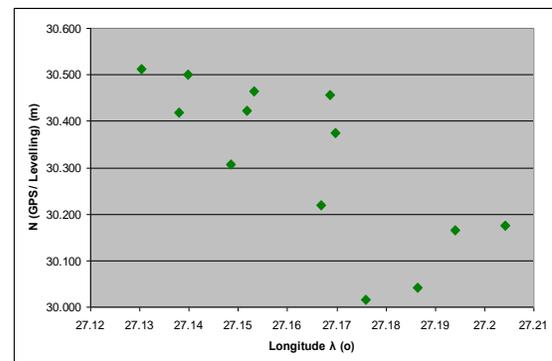
Also by using the formula $\Delta N_{ij} = \Delta H_{ij} - \Delta h_{ij}$, the 28 geoid undulation differences are

calculated and the undulation N at each BM is determined by another least square adjustment with $\sigma_0 = \pm 11$ mm. The geoid undulations N_i range between 30.01m to 30.50m.

Table 1 summarizes the results. In figure 2 the N/S (2a) and the W/E (2b) profiles of the geoid undulation are presented, while the contours of the geoid undulation N are drawn by 5cm interval (figure 3).It is obvious that there is a regular tilt of the geoid surface towards the ellipsoid from S/E to N/W equal to 0.07‰.



(a)



(b)

Figure 2: The profile of the geoid undulation N/S (2a) and W/E (2b)

Table 1: The positions of the benchmarks and the adjustments results

point	ϕ	λ	H (m)	h (m)	N (m)
T1	36°.62067262	27°.16857822	27.987	58.443	30.456
T2	36°.61470111	27°.13980346	2.123	32.623	30.500
G1	36°.61319996	27°.15322208	3.239	33.704	30.465
G2	36°.60193776	27°.15182674	259.919	290.342	30.423
G3	36°.59729204	27°.20420953	3.173	33.349	30.175
G4	36°.57876786	27°.16688977	112.615	142.835	30.220
G5	36°.58912929	27°.19411270	277.099	307.265	30.166
G6	36°.56817932	27°.18635222	273.709	303.750	30.041
G7	36°.55906276	27°.17591012	33.808	63.822	30.015
G8	36°.57710666	27°.14853694	280.835	311.141	30.306
G9	36°.59150701	27°.13803597	252.468	282.886	30.418
KASTRO	36°.60615829	27°.13030798	112.374	142.887	30.513
3010	36°.60517342	27°.16971229	453.807	484.182	30.375
	<i>Mean uncertainty</i>		$\pm 6mm$	$\pm 10mm$	$\pm 11mm$

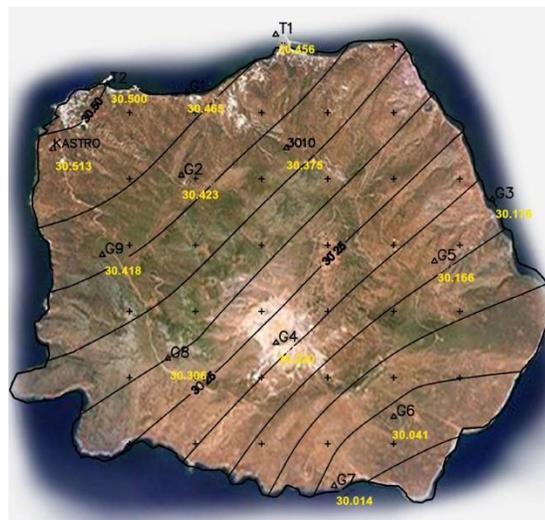


Figure 3: The Nisyros' geoid contours by 5cm interval

IV. PARAMETRIC APPROXIMATION MODELS

Polynomial regression method was applied in order to express the geoid surface via an adaptation equation, as the examined area is small. First and second degree equations were tried. The following equation 4 presents a Simple Planar Surface with RMSE ±2.3 cm, as the equation 5 presents a better approximation by second degree polynomial with RMSE ±0.4 cm.

$$N_i(m) = a_0 + a_1 \cdot (\varphi_i - \varphi_0) + a_2 \cdot (\lambda_i - \lambda_0) = 30.31 + 6.1662 \cdot (\varphi_i - \varphi_0) - 4.0684 \cdot (\lambda_i - \lambda_0) \tag{4}$$

$$N_i(m) = a_0 + a_1 \cdot (\varphi_i - \varphi_0) + a_2 \cdot (\lambda_i - \lambda_0) + a_3 \cdot (\varphi_i - \varphi_0) \cdot (\lambda_i - \lambda_0) + a_4 \cdot (\varphi_i - \varphi_0)^2 + a_5 \cdot (\lambda_i - \lambda_0)^2 = 30.34 + 5.775 \cdot (\varphi_i - \varphi_0) - 4.103 \cdot (\lambda_i - \lambda_0) + 33.813 \cdot (\varphi_i - \varphi_0) \cdot (\lambda_i - \lambda_0) - 33.813 \cdot (\varphi_i - \varphi_0)^2 - 10.205 \cdot (\lambda_i - \lambda_0)^2 \tag{5}$$

Where φ_0, λ_0 (rad) are the mean values of latitude and longitude correspondingly Figure 4 illustrates the geoid surface modeling by using Simple Planar Surface (left) and Quadratic Surface (right).

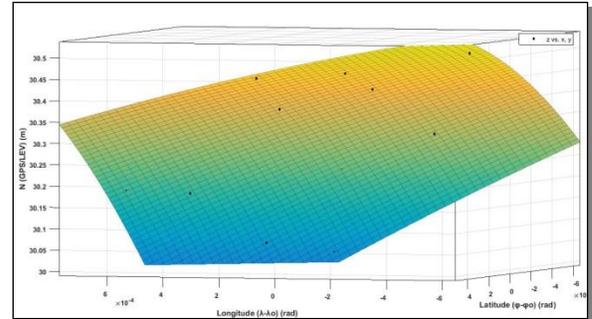
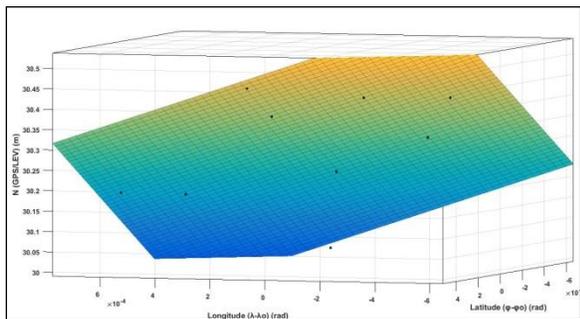


Figure 4: The Geoid surface modeling by Simple Planar (left) and Quadratic Surface (right)

V. A REDUCTION EQUATION OF N^{EGM08}

As it is referred above Nisyros is a limited almost round area of about 4Km radius. The resolution of EGM08 reaches the 8Km. So, there is a little likelihood to derive reliable N^{EGM08} values for the BMs. Nevertheless in order to exploit the capability of EGM08 it was decided to carry out the procedure and also to attempt the determination of a reduction equation in order to correct the provided N^{EGM08} .

The geometric heights of the BMs are calculated in the zero-tide system according to the EGM2008 global geopotential model on the GRS80 reference ellipsoid¹ by using the harmonic_synth_v02 software program that is freely provided by the NGA/EGM development team[40].

The zero degree term, N_0 which represents the difference between the GM-values of the EGM2008 mean ellipsoid and the

correspondent ones of the reference ellipsoid GRS80 of GPS/ leveling system was also included. The additive term N_o is given by the equation 6 as the contribution of the zero-degree harmonic to the EGM2008 geoid height with respect to the GRS80 ellipsoid [41].

$$N_o(m) = \frac{GM - GM'}{R \cdot \gamma} - \frac{W_o - U_o}{\gamma} \quad (6)$$

where

GM' and U_o represents the Somigliana - Pizzeti normal gravity field referred to the GRS80 ellipsoid ($GM' = 398600.50 \times 10^9 \text{ m}^3/\text{s}^2$ and $U_o = 62636860.85 \text{ m}^2/\text{s}^2$).

The Earth's geocentric gravitational constant $GM = 398600.4415 \times 10^9 \text{ m}^3/\text{s}^2$ and the geoidal gravity potential equals to $W_o = 62636856.00 \text{ m}^2/\text{s}^2$.

The value $R = 6371008.771 \text{ m}$ was set for the mean Earth radius according to GRS80 and finally the normal gravity γ on the reference ellipsoid was computed at each point from Somigliana's formula.

According to the above mentioned values the zero-degree term N_o is incorporated to the computations equal to -0.422 m for all the BMs due to the limited area of the island. Table 2 presents the N^{EGM08} as well as the differences ΔN between $N^{\text{GPS/LEV}}$ and N^{EGM08} , which range between -50 cm to -58 cm with mean value of $-0.531 \text{ m} \pm 24 \text{ mm}$.

In figure 5 the profile of the differences ΔN towards N/S and W/E is illustrated.

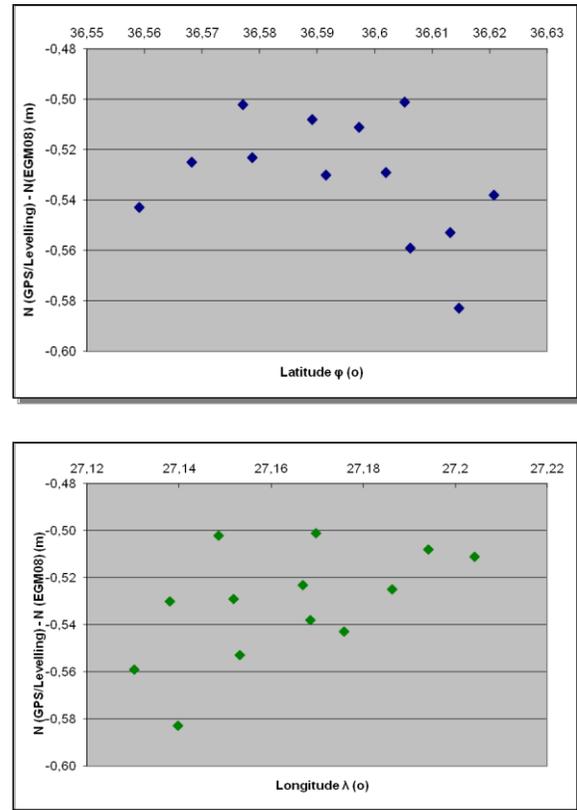


Figure 5: The profile of the differences ΔN towards N/S and W/E

Table 2: N^{EGM08} values and the differences ΔN between $N^{GPS/LEV}$ and N^{EGM08}

Point	N^{EGM08}	$N^{GPS/LEV} - N^{EGM08}$
T1	30.994	-0.538
T2	31.083	-0.583
G1	31.018	-0.553
G2	30.952	-0.529
G3	30.686	-0.511
G4	30.743	-0.523
G5	30.674	-0.508
G6	30.566	-0.525
G7	30.558	-0.543
G8	30.808	-0.502
G9	30.948	-0.530
KASTRO	31.072	-0.559
3010	30.876	-0.501
Mean value		-0.531m ±24mm

In order to provide a reduction equation for the exploitation of N^{EGM08} values a polynomial regression is performed. The first order equation 7 is calculated with RMSE ±2cm, as a quadratic surface (equation 8) approximates better the area with RMSE±1cm. Namely equation 8 can improve the $N^{EGM2008}$ values, up to 60%.

$$\Delta N_i(m) = -0.531 - 0.275 \cdot (\varphi_i - \varphi_0) - 0.508 \cdot (\lambda_i - \lambda_0) \quad (7)$$

$$\Delta N_i(m) = -0.51 - 0.602 \cdot (\varphi_i - \varphi_0) + 0.504 \cdot (\lambda_i - \lambda_0) + 41.37 \cdot (\varphi_i - \varphi_0) \cdot (\lambda_i - \lambda_0) - 31.22 \cdot (\varphi_i - \varphi_0)^2 - 11.62 \cdot (\lambda_i - \lambda_0)^2 \quad (8)$$

Where φ_0, λ_0 (rad) are the mean values of latitude and longitude correspondingly .

Finally figure 6 illustrates the approximation of the differences ΔN on the area by using

Simple Planar Surface (left) and Quadratic Surface (right)

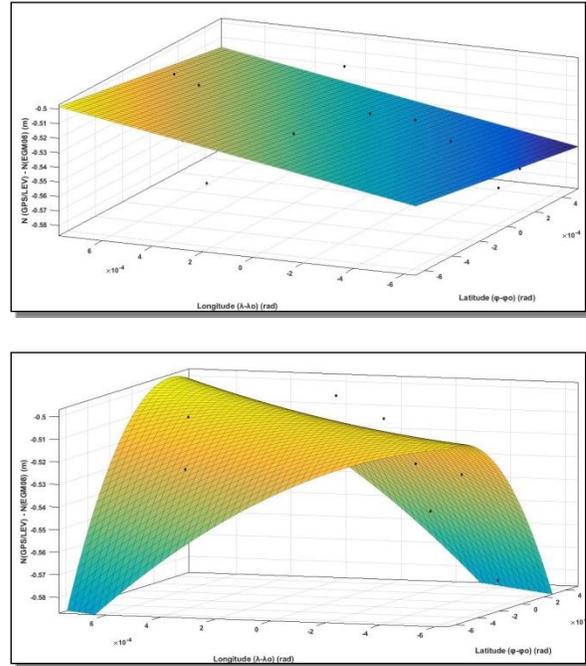


Figure 6: The approximation of the differences ΔN by using Simple Planar Surface (up) and Quadratic Surface (down)

VI. ZERO-HEIGHT GEOPOTENTIAL LEVEL DETERMINATION

The unification of the different height datums is of major significance for countries like Greece, where decades of islands referred to different vertical systems namely to different zero-height geopotential level surfaces. Today the presence of a global geopotential model where a reference zero equipotential surface of Earth's gravity field is defined as W_0 , gives the opportunity to link local vertical zero-height surfaces

W_o^{LVD} , by calculated their differences from the global W_o .

Various methods have been used for the estimation of the fundamental parameter W_o^{LVD} , which can be broadly classified into two basic categories described by Kotsakis et al. (2010). The first one is based on the combined adjustment of GGM and GPS/leveling data [42], [43], while the other uses the formulation of a geodetic boundary value problem with the use of gravity anomaly data over different LVD zones [44], [45], [46], [47]. According to the first method W_o^{LVD} of the Local Vertical Datum was computed using the equation 9 [42], [48]:

$$W_o^{LVD} = W_o - \frac{\sum_i^m (h_i - H_i - N_i) g_i}{m} \quad (9)$$

where

$$W_o = 62\,636\,856.0 \text{ m}^2/\text{s}^2$$

h_i = the geometric height of each BM derived by GNSS measurements

H_i = the orthometric height of each BM

N_i = the geoid height derived from GGMs

g_i = the gravity at each BM computed from GGMs and

m = the total number of the available BMs i.e 13 in this case.

Three parametric models have been tested according to Kotsakis et al. 2012. The first

one is the null model, where no systematic errors or biases are modeled. The second one, where systematic differences are modeled in terms of a scaling factor and the third one, where the tilt between the two reference surfaces, the geoid and the ellipsoid, is represented by the tilt components x_1 for N/S and x_2 for the W/E. Table 3 illustrates the results. The W_o^{LVD} which came out by the three models are close to each other while the first model provides the smaller RMSE ± 4 cm.

Table 3: The estimated W_o^{LVD} for Nisyros by using three different parametric models.

Model $a_i^T \cdot X$	W_o^{LVD} (m^2/s^2)	$W_o^{LVD} - W_o$ (m^2/s^2)	δH_o^{LVD} (cm)
0	62636861.20 ± 0.04	5.20 ± 0.04	-53.1 ± 4
$\delta s \cdot H_i$	62636861.37 ± 0.07	5.37 ± 0.07	-54.8 ± 7
$x_1 \cdot (\varphi_i - \varphi_0) + x_2 \cdot (\lambda_i - \lambda_0) \cdot \cos \varphi_i$	62636861.21 ± 0.06	5.21 ± 0.06	-53.2 ± 6

Finally figure 7 illustrates the vertical shifts in (cm) of the LVDs in three neighboring Hellenic islands and the mainland (Piraeus) relative to the conventional global reference value W_o as they have been calculated by Kotsakis et al 2012.

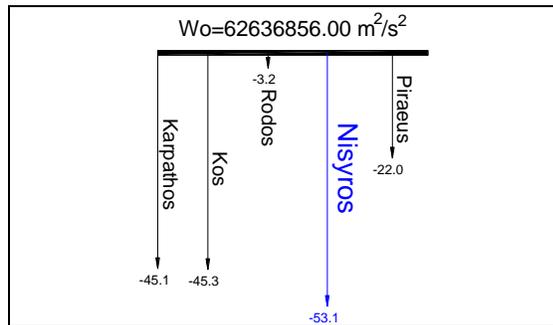


Figure 7: Vertical shifts of the LVDs in neighboring to Nisyros Hellenic islands and Piraeus

VII. CONCLUSION

The thirteen benchmarks that were established on Nisyros Island are distributed at an area grid of 1km to 4Km and therefore can support any infrastructure work of sub cm accuracy.

The accurate trigonometric heighting method is proved effective providing the orthometric heights of the BMs by ± 6 mm.

The geoid heights NGPS/LEV that were calculated by the network adjustment ranges between 30.01m to 30.50m with accuracy ± 1.1 cm. Hence there is a variation of 50cm from SE to NW equal to 0.07%. The polynomial regression by a quadratic surface models Nisyros' geoid with RMSE ± 6 mm.

The differences ΔN between N GPS/LEV and NEGM08 ranges between -50cm to -58cm. So a quadratic reduction equation is determined in order to correct the EGM08 provided N by accuracy of ± 1 cm. Thus the EGM08 is now exploitable for Nisyros.

As the unification of the zero height level surfaces is the main request for the Hellenic Islands and the mainland, the W_0^{LVD} of the LVD is calculated by using three different models which provide similar results. The reference of the Zero-height equipotential surface is found to be -53.1cm lower than the W_0 of the EGM08. Also this value is lower than those obtained for neighboring to Nisyros Hellenic islands.

Consequently, a precise local geoid definition of Nisyros Island is implemented by cm accuracy. Simultaneous measurements of orthometric and geometric height differences were carried out within two weeks time for an area of 41 Km². So about 3 Km² per day are covered. The 13 BMs which were established and the 28 connections, it means 0.3 BM per Km² and 0.7 connections per Km², are proved adequate for the local geoid definition.

The above experimental application could be a reference pilot project for the time and the cost that are demanded and the accuracy that can be achieved for larger areas by using the GPS/leveling procedure.

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